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Effect of the acceleration current on the centrifugal interchange instability

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[1] The centrifugal interchange instability is the primary driver of radial plasma transport in the magnetospheres of Jupiter and Saturn. Most previous theoretical treatments of this instability have ignored the role of the acceleration current and have assumed that the divergence of the centrifugal drift current in the magnetosphere is closed by Pedersen currents in the underlying ionosphere, through the connection provided by Birkeland (magnetic-field-aligned) currents. This is a reasonable approximation when the radial transport speed is small compared with the rotation speed. However, the exponential growth of the instability inevitably leads to the eventual violation of this condition. I analyze a simplified model of the Io plasma torus to show that when the radial transport speed becomes comparable to the rotation speed, the acceleration current becomes the primary mechanism for closure (actually local cancellation) of the centrifugal drift current, and the connection to the planetary ionosphere therefore becomes irrelevant. An immediate consequence is that the growth rate of the instability does not exceed the planetary rotation rate.

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1. Introduction

[2] The magnetospheres of Jupiter and Saturn have rapid rotation rates and strong internal plasma sources, the two main ingredients for producing the centrifugal interchange instability. Interchange motions, as originally defined by *Gold* [1959], do not alter the magnetic-field configuration, so their stability is determined solely by the radial distribution of plasma mass density and pressure. In the region outside a source, where the gradient of flux-tube content is inward, the distribution is unstable. Subsequent work has generalized the interchange concept to include the effects of magnetic-field variations, which can become important when β (ratio of plasma to magnetic field pressure) is not $\ll 1$ [*Cheng*, 1985; *Southwood and Kivelson*, 1987; *Ferrière et al.*, 1999]. The principal focus of this paper is the Io plasma torus, where $\beta \ll 1$ and the simple Gold criterion is sufficient.

[3] In gasdynamic terms, the mass distribution produced by the internal source is unstable to interchange motions because the centrifugal force of corotation exceeds the inward force of planetary gravity [e.g., *Gold*, 1959; *Siscoe and Summers*, 1981; *Southwood and Kivelson*, 1987]. In electrodynamic terms [e.g., *Siscoe and Summers*, 1981; *Huang and Hill*, 1991; *Yang et al.*, 1994; *Pontius et al.*, 1998], any azimuthal variation of plasma mass density ρ is amplified because it produces a divergence of the centrifugal drift current

$$\mathbf{j}_{cent} = \left(\rho\Omega^2 r/B\right)\hat{\varphi} \quad (1)$$

which is closed by ionospheric Pedersen currents. Current closure requires an electric field whose $\mathbf{E} \times \mathbf{B}$ drift amplifies the radial displacement that led to the divergence. Thus both gasdynamic and electrodynamic arguments indicate that the mass distributions in the magnetospheres of Jupiter and Saturn are centrifugally unstable. In (1), (r, φ, z) is a cylindrical coordinate system aligned with the planetary spin vector $\mathbf{\Omega}$, and B is the strength of the magnetic field in the equatorial plane, assumed to be in the $-z$ direction.

[4] Inclusion of the plasma thermal energy density in the gasdynamic analysis, or, equivalently, of the gradient-curvature drift current in the electrodynamic analysis, reinforces this conclusion. The plasma distribution from an internal source is still unstable outside the source, but the instability is describable, in part, as a pressure-driven (“flute”) instability rather than a centrifugal instability. For Jupiter, in the neighborhood of the Io plasma torus, the thermal energy density is perhaps 30% of the centrifugal potential energy density [e.g., *Huang and Hill*, 1991]. In this paper I will neglect the thermal energy density and hence the gradient-curvature drift currents; their inclusion would modify the results quantitatively but not qualitatively.

[5] In a rotation-dominated magnetosphere, the magnetospheric plasma is centrifugally confined near the equatorial plane [*Hill and Michel*, 1976; *Siscoe*, 1977; *Vasyliunas*,

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1983], and the equation of motion, integrated across the equatorial plasma sheet, is

$$\eta \frac{d\mathbf{v}}{dt} = \eta\Omega^2\mathbf{r} - 2\eta\Omega\hat{\mathbf{z}} \times \mathbf{v} + \hat{\mathbf{z}} \times \mathbf{J} \qquad (2)$$

where

$$\eta \equiv \int \frac{\rho dz}{B} \qquad (3)$$

is the mass per unit magnetic flux and **J** is the current density integrated across the sheet. The velocity **v** is measured in the corotating frame of reference; thus the three terms on the right side of (2) are the centrifugal, Coriolis, and Lorentz forces, all per unit magnetic flux. The cross product of $\hat{\mathbf{z}}$ with (2) gives

$$\mathbf{J} = \eta\Omega^2 r\hat{\varphi} + 2\eta\Omega\mathbf{v} - \eta\hat{\mathbf{z}} \times \frac{d\mathbf{v}}{dt} \qquad (4)$$

where the three terms on the right can be identified as the centrifugal current (1) integrated across the sheet, the Coriolis current, and the acceleration current. *Hill* [1983] has noted that these three terms are of zeroth, first, and second order, respectively, in the ratio $v/\Omega r$ (where the last part of the statement follows from the ansatz $d\mathbf{v}/dt \sim \mathbf{v} \bullet \nabla\mathbf{v}$).

[6] It is therefore justifiable to neglect the last two terms, provided that attention is restricted to situations for which $v \ll \Omega r$. This has been done in most previous treatments of the centrifugal interchange instability, both analytical [*Siscoe and Summers*, 1981; *Southwood and Kivelson*, 1987; *Huang and Hill*, 1991] and numerical [*Yang et al.*, 1992, 1994; *Pontius et al.*, 1998]. However, recent numerical simulations with the RCM-J (Rice Convection Model for Jupiter) indicate that this condition is quickly violated, even within the confines of the Io plasma torus, because the dominant scale size of the interchange cells is very small, corresponding to very large linear growth rates [*Spiro et al.*, 2000; *Goldstein et al.*, 2001, 2002]. We are thus motivated to consider the effects of the Coriolis and acceleration currents on the development of the centrifugal interchange instability within the Io plasma torus.

[7] Work is in progress to include these effects within the RCM-J formalism. The Coriolis effect can be included relatively simply by the use of an effective Hall conductance in the Jovian ionosphere. Inclusion of the acceleration current, however, is more complicated and requires a major reworking of the numerical code. In the meantime, as a prelude to this numerical work, it seems useful to consider the effects of the acceleration current through a simple, analytically tractable model, and that is the purpose of this paper.

[8] There are two notable exceptions to the above rule (neglect of the Coriolis and acceleration terms in the equation of motion). *Vasyliunas* [1994] has elucidated the role of the acceleration timescale $\tau_a = \eta/\Sigma B$, i.e., the timescale required for the Jovian ionosphere to impose its motion on the magnetospheric plasma, where Σ is the height-integrated ionospheric Pedersen conductivity. He shows that when $\Omega\tau_a > \sim 1$ (generally true in the Jovian magnetosphere beyond $r \sim 10\ R_J$), the growth rate of the centrifugal interchange instability is limited to values $\gamma < \sim\Omega$. *Pontius* [1997] has generalized this result to include the effect of the Coriolis acceleration, which reduces the effective value of Ω for outflowing plasma. He finds a more stringent limit on the growth rate, particularly for values of $\Omega\tau_a \gg 1$. Both of these papers simplify the ionosphere-magnetosphere coupling equation by assuming that the ionospheric current is equal in magnitude and opposite in direction to the magnetospheric current when mapped along the field. Here I relax that assumption and recover essentially the same result, that the growth rate is limited to values $< \sim\Omega$. This limitation becomes important even within the Io torus ($r \sim 6\ R_J$) for sufficiently small scale sizes.

2. Model

[9] In order to obtain an analytically tractable model, I adopt the simplest possible representation of a plasma torus that retains the essential physics of the centrifugal interchange instability. Specifically, I assume a step-function distribution of the plasma mass content per unit magnetic flux:

$$\eta(L, \varphi, t) = \begin{Bmatrix} \eta_1, & L < L_1(\varphi, t) \\ 0, & L > L_1(\varphi, t) \end{Bmatrix} \qquad (5)$$

where $L = r/R_J$ and $L_1(\varphi, t)$ is the outer edge position. I assume that the outer edge is perturbed by a simple sinusoidal ripple:

$$L_1(\varphi, t) = L_0 + \delta L_0 \sin(m\varphi)e^{\gamma t} \qquad (6)$$

where L_0 ($\approx$6) and δL_0 are given constants, m is an integer, and $\gamma(m)$ is a real growth rate to be determined.

[10] I assume a spin-aligned dipole magnetic field and a simple ionosphere with a uniform height-integrated Pedersen conductivity Σ in each hemisphere. The height-integrated Hall conductivity is also assumed to be uniform, in which case its value does not matter because the Hall current does not diverge. Then the ionosphere-magnetosphere coupling equation, expressed in equatorial (L, φ) coordinates, is

$$\frac{\partial}{\partial L}(LJ_L) + \frac{\partial J_\varphi}{\partial \varphi} = \frac{\Sigma}{R_J}\left\{4\frac{\partial}{\partial L}\left[L\left(1 - \frac{1}{L}\right)^{1/2}\frac{\partial \Phi}{\partial L}\right] + \frac{1}{L}\left(1 - \frac{1}{L}\right)^{-1/2}\frac{\partial^2 \Phi}{\partial \varphi^2}\right\} \qquad (7)$$

where $\mathbf{J} = (J_L, J_\varphi)$ is the surface current vector in the equatorial current sheet, and Φ is the electrostatic potential (assumed to be a field-line invariant). This is derived, for example, in the appendix of *Hill et al.* [1981] and (in terms of Euler-potential coordinates α, β) in Appendix A of *Huang et al.* [1990] and in equation (45) of *Ferrière and Blanc* [1996]. (There is a misprint in equation (A11) of *Hill et al.* [1981], where the second term on the right has an exponent of the wrong sign. Also, when transforming from the (α, β) coordinates of Huang et al. to the (L, φ)

coordinates used here, one has to allow for the fact that Huang et al. implicitly defined Σ to be the conductivity integrated along the ionospheric portion of a field line, rather than integrated over altitude.)

[11] To obtain an analytically tractable problem, I adopt the ‘‘flat-Jupiter’’ approximation

$$\left(1-\frac{1}{\mathrm{L}}\right)^{1/2} \approx \text{ constant} \sim 1 \tag{8}$$

so (7) simplifies to

$$\frac{4}{L}\frac{\partial}{\partial L}\left(L\frac{\partial\Phi}{\partial L}\right)+\frac{1}{L^2}\frac{\partial^2\Phi}{\partial\varphi^2}=\frac{R_J}{\Sigma}\left[\frac{1}{L}\frac{\partial}{\partial L}(L\,J_L)+\frac{1}{L}\frac{\partial J_\varphi}{\partial\varphi}\right] \tag{9}$$

Apart from the factor 4 on the left (which results from the dipole mapping), (9) is a Poisson equation for the potential $\Phi\,(L, \varphi, t)$ with a source term proportional to the divergence of the magnetospheric current $\mathbf{J}(L, \varphi, t)$.

[12] $\mathbf{J}(L, \varphi, t)$ is, of course, not given, but depends on $\Phi\,(L, \varphi, t)$ through the magnetospheric equation of motion (4). Adopting the quasi-static MHD approximation

$$\mathbf{v}=\frac{\mathbf{B}\times\nabla\Phi}{B^2}=-\frac{1}{B}\hat{\mathbf{z}}\times\nabla\Phi \tag{10}$$

(4) can be written

$$\mathbf{J}=\eta\Omega^2 r\hat{\varphi}-\frac{2\eta\Omega}{B}\hat{\mathbf{z}}\times\nabla\Phi-\frac{\eta}{B}\frac{\partial}{\partial t}(\nabla\Phi) \tag{11}$$

The second (Coriolis) term on the right-hand side will be neglected here. Because this current contribution is parallel to $\mathbf{v}$, its $\mathbf{J}\times\mathbf{B}$ force deflects the flow sideways but does not accelerate or decelerate it. Then (11) becomes

$$J_L=-\frac{\eta}{R_J B_J}L^3\frac{\partial}{\partial L}\left(\frac{\partial\Phi}{\partial t}\right) \tag{12a}$$

$$J_\varphi=\eta\Omega^2 R_J L-\frac{\eta}{R_J B_J}L^2\frac{\partial}{\partial\varphi}\left(\frac{\partial\Phi}{\partial t}\right) \tag{12b}$$

and using these in (9) gives

$$\frac{4}{L}\frac{\partial}{\partial L}\left(L\frac{\partial\Phi}{\partial L}\right)+\frac{1}{L^2}\frac{\partial^2\Phi}{\partial\varphi^2}=\frac{\Omega^2R_J^2}{\Sigma}\frac{\partial\eta}{\partial\varphi}-\frac{1}{\Sigma B_J L}\frac{\partial}{\partial L}\cdot\left[\eta L^4\frac{\partial}{\partial L}\left(\frac{\partial\Phi}{\partial t}\right)\right]-\frac{L}{\Sigma B_J}\frac{\partial}{\partial\varphi}\left[\eta\frac{\partial}{\partial\varphi}\left(\frac{\partial\Phi}{\partial t}\right)\right] \tag{13}$$

[13] I now assume that $\Phi(L, \varphi, t)$ is separable in the form

$$\Phi(L,\varphi,t)=\Phi_m(L)\exp[i(m\varphi-\omega t)]=\Phi_m(L)\exp(im\varphi+\gamma t) \tag{14}$$

where $\gamma = -i\omega$, and I anticipate the result that the solution of interest is a standing wave in the corotating frame with a purely imaginary frequency, i.e., a purely real growth rate [e.g., *Huang et al.*, 1990; *Huang and Hill*, 1991; *Yang*, 1992; *Yang et al.*, 1994]. This assumes that an initially sinusoidal perturbation will remain sinusoidal (as in (6)), so it cannot capture the nonlinear evolution of the interchange cells, but it should capture their initial linear growth. Comparison of (6) and (14) indicates that the potential $\Phi(\varphi)$ has extrema at the null longitudes of the boundary displacement $L_1(\varphi)$, as required by symmetry. Then (13) becomes

$$\left(1+\frac{\gamma\tau_a}{4}\right)\frac{d^2\Phi_m}{dL^2}+(1+\gamma\tau_a)\frac{1}{L}\frac{d\Phi_m}{dL}-(1+\gamma\tau_a)\frac{m^2}{4L^2}\Phi_m(L)=-\frac{\gamma\tau_a}{4}\left(\frac{1}{\eta}\frac{\partial\eta}{\partial L}\right)\frac{d\Phi_m}{dL}+\frac{1}{4}\left(\frac{1}{\eta}\frac{\partial\eta}{\partial\varphi}\right)\cdot\left[\Phi_0 e^{-im\varphi-\gamma t}-\frac{im}{L^2}\gamma\tau_a\Phi_m(L)\right] \tag{15}$$

where

$$\tau_a\equiv\frac{\eta L^3}{\Sigma B_J}\sim(0.43\ hr)\left(\frac{1\mathrm{S}}{\Sigma}\right)\left(\frac{\eta}{\eta_1}\right)\left(\frac{L}{6}\right)^3 \tag{16}$$

and

$$\Phi_0\equiv\frac{\Omega^2R_J^2\eta}{\Sigma}\sim(475\ \mathrm{kV})\left(\frac{1\mathrm{S}}{\Sigma}\right)\left(\frac{\eta}{\eta_1}\right) \tag{17}$$

The numerical estimates in (16) and (17) are based on the value $\eta_1 \sim 3 \times 10^{-3}$ kg/W, which is typical of the torus density peak (number density $\sim$2000/cm^3, mean ion mass $\sim$24 amu, vertical scale height $\sim$1 R_J), and on the standard values R_J = 71,400 km, B_J = 4.28 G, and $\Omega = 1.76 \times 10^{-4}$/s. Note that τ_a (16) is exactly the acceleration time defined by *Vasyliunas* [1994]. It is interesting to note that Φ_0 (17) is comparable to the corotational potential drop across Io's diameter; this appears to be a coincidence, at least at the present level of understanding.

[14] Both τ_a and Φ_0 are proportional to η and are hence discontinuous at $L = L_1(\varphi, \mathrm{t})$, dropping from constant values (given by $\eta = \eta_1$) for $L < L_1(\varphi, \mathrm{t})$ to zero for $L > L_1(\varphi, \mathrm{t})$. Thus (15) is an ordinary differential equation for $\Phi_m(L)$ except at the discontinuity, where it also has a (φ, t) dependence. The source terms on the right-hand side are zero except at the discontinuity. Outside the discontinuity, we have $\eta = 0$, hence $\tau_a = 0$, and (15) becomes

$$\frac{d^2\Phi_m}{dL^2}+\frac{1}{L}\frac{d\Phi_m}{dL}-\frac{m^2}{4L^2}\Phi_m(L)=0 \tag{18}$$

which has the general solution

$$\Phi_m(L)=\Phi_+L^{m/2}+\Phi_-L^{-m/2} \tag{19}$$

The condition of good behavior at $L \to \infty$ gives $\Phi_+ = 0$, and the condition of continuity at $L = L_1$ gives

$$\Phi_-=\Phi_m(L_1)L_1^{m/2} \tag{20}$$

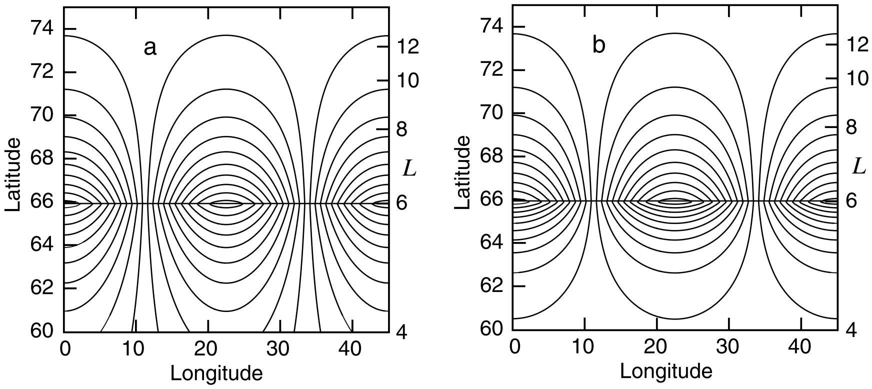

Figure 1. Potential mode structure mapped to a rectangular ionospheric longitude-latitude grid, with L value indicated on the right vertical axis, for azimuthal wave number $m = 8$. (a) The case $\gamma\tau_a \ll 1$, from (14) and (23). (b) The case $\gamma\tau_a \gg 1$, from (14) and (35). The two solutions are identical for $L > L_1 = 6$.

so the solution of (15) for $L > L_1(\varphi, t)$ is

$$\Phi_m(L) = \Phi_m(L_1)\left(\frac{L}{L_1}\right)^{-m/2} \qquad (L > L_1(\varphi, t)) \tag{21}$$

The solution for $L < L_1(\varphi, t)$ is complicated by the fact that τ_a is a function of L (16), and I have found no analytic solution for the general case $\gamma\tau_a \sim 1$. There are, however, two easily solved limiting cases of interest, namely $\gamma\tau_a \ll 1$ and $\gamma\tau_a \gg 1$.

2.1. Limiting Case $\gamma\tau_a \ll 1$

[15] This case corresponds to ignoring the acceleration current, the third term in (11), as in most previous work. Ignoring this term is equivalent to setting $\gamma\tau_a = 0$ in (15), so the general solution is again (19), but now the condition of good behavior at $L \to 0$ gives $\Phi_- = 0$, and the condition of continuity at $L = L_1$ gives

$$\Phi_+ = \Phi_m(L_1)L_1^{-m/2} \tag{22}$$

so the complete solution is

$$\Phi_m(L) = \Phi_m(L_1)\left\{\begin{array}{ll} \left(\dfrac{L}{L_1}\right)^{m/2}, & L < L_1 \\ \left(\dfrac{L}{L_1}\right)^{-m/2}, & L > L_1 \end{array}\right\} \tag{23}$$

in agreement with equation (15) of *Huang and Hill* [1991]. The potential structure ((14) with (23)) is illustrated in Figure 1a for $m = 8$. We can determine $\Phi_m(L_1)$ by integrating (15) (for $\gamma\tau_a = 0$) across the discontinuity at $L = L_1$. To do this, we need an expression for $\partial\eta/\partial\varphi$ at $L = L_1$, which is obtained from (5) and (6):

$$\eta = \eta_1\left[1 - H\left(L - L_0 + i\delta L_0 e^{im\varphi+\gamma t}\right)\right] \tag{24}$$

where H is the Heaviside (unit step) function. Thus

$$\frac{\partial\eta}{\partial\varphi} = m\eta_1\delta L_0 e^{im\varphi+\gamma t}\delta[L - L_1(\varphi, t)] \tag{25}$$

where δ is the Dirac delta function. Using this in (15) and integrating across the discontinuity gives

$$\left[\frac{d\Phi_m}{dL}\right]_{L_1+} - \left[\frac{d\Phi_m}{dL}\right]_{L_1-} = -\frac{m}{L_1}\Phi_m(L_1) = \frac{m\eta_1\omega^2 R_J^2\delta L_0}{4\Sigma} \tag{26}$$

or

$$\Phi_m(L_1) = -\frac{\eta_1\omega^2 R_J^2 L_1\delta L_0}{4\Sigma}\left(= -\frac{\Phi_0 L_1\delta L_0}{4}\right) \tag{27}$$

Finally, the growth rate γ can be obtained by combining

$$\frac{dL_1}{dt} = -i\gamma\delta L_o e^{im\varphi+\gamma t} \tag{28}$$

from (6), with

$$R_J\frac{dL_1}{dt} = \frac{L_1^3}{B_J}\frac{1}{R_J L_1}\left(\frac{\partial\Phi}{\partial\varphi}\right)_{L_1} \tag{29}$$

from (10), using (14), (23), and (27). This gives

$$\gamma = m\frac{\eta_1\Omega^2 L_1^3}{4\Sigma B_J} \tag{30}$$

in agreement with equation (22) of *Huang and Hill* [1991] and equation (15) of *Yang et al.* [1994]. (There is a second eigenvalue $\gamma = 0$, which is of no physical interest.)

[16] Thus a purely sinuisoidal perturbation of a single outer edge should grow exponentially in time at the rate given by (30). Numerically, if $\eta_1 = 3 \times 10^{-3}$ kg/W, $\Sigma = 1$ S, and $L_1 = 6$, then $\gamma \approx m\Omega/15 \approx m/(24$ hours). This is identical to the analytic results of *Huang and Hill* [1991] and *Yang et al.* [1994], and consistent with the simulation results of *Yang et al.* [1994], who used a more complicated multiedged (and therefore less unstable) initial condition with $m = 8$, and found a somewhat slower growth during the

linear phase. This result for $\gamma\tau_a \ll 1$ is not new; it is, however, needed to establish the context for the result to follow in the next subsection for $\gamma\tau_a \gg 1$. It is interesting to note that this allegedly linear analysis gives a linear growth rate (30) that is itself an exponentially growing function of time (because L_1 is). This indicates that nonlinearity, when it sets in, should do so with a vengeance.

2.2. Limiting Case $\gamma\tau_a \gg 1$

[17] This case, as will become evident below, corresponds to assuming that the acceleration current, the third term in (11), just balances the centrifugal current, the first term in (11). Ignoring unity compared to $\gamma\tau_a$, (15) becomes

$$\frac{d^2\Phi_m}{dL^2} + \frac{4}{L}\frac{d\Phi_m}{dL} - \frac{m^2}{L^2}\Phi_m(L) = -\left(\frac{1}{\eta}\frac{\partial\eta}{\partial L}\right)\frac{d\Phi_m}{dL} + \left(\frac{1}{\eta}\frac{\partial\eta}{\partial\varphi}\right)\left[\frac{\Phi_0}{\gamma\tau_a}e^{-im\varphi-\gamma t} - \frac{im}{L^2}\Phi_m(L)\right] \quad (31)$$

(Note that just because $1/\gamma\tau_a$ is neglected compared to unity, this does not mean that the $\Phi_0/\gamma\tau_a$ term in (31) can be neglected compared to the term next to it inside the brackets. Indeed, it will become apparent below that ignoring this term would give a nonsense result.) Again, the terms on the right-hand side are zero except at the discontinuity, so the general solution to (31) in this case, for $L < L_1$, is

$$\Phi_m(L) = \Phi_+ L^{p+} + \Phi_- L^{p-} \quad (32)$$

where

$$p_\pm = \frac{3}{2}\left(\pm\sqrt{\frac{4m^2}{9}+1} - 1\right) \approx \pm m (m \gg 1) \quad (33)$$

Good behavior at L $\rightarrow$ 0 requires $\Phi_- = 0$, and continuity at $L = L_1$ gives

$$\Phi_+ = \Phi_m(L_1)L_1^{-m} \quad (34)$$

so the full solution for this case ($\gamma\tau_a \gg 1$) is

$$\Phi_m(L) \approx \Phi_m(L_1)\left\{\begin{array}{ll}\left(\frac{L}{L_1}\right)^m, & L < L_1 \\ \left(\frac{L}{L_1}\right)^{-m/2}, & L > L_1\end{array}\right\} \quad (35)$$

[18] The potential structure ((14) with (35)) is illustrated in Figure 1b for $m = 8$. It is, of course, identical to Figure 1a for $L > L_1$, and only quantitatively different for $L < L_1$. As before, $\Phi_m(L_1)$ is determined by integrating (31) across the discontinuity at $L = L_1$. Before doing this, however, it is helpful to multiply (31) by $\eta(L)$:

$$\frac{\partial}{\partial L}\left(\eta\frac{d\Phi_m}{dL}\right) + \frac{4\eta}{L}\frac{d\Phi_m}{dL} - \frac{m^2\eta}{L^2}\Phi_m(L) = \frac{\partial\eta}{\partial\varphi}\left[\frac{\Phi_0}{\gamma\tau_a}e^{-im\varphi-\gamma t} - \frac{im}{L^2}\Phi_m(L)\right] \quad (36)$$

which can be integrated, using (25), to obtain

$$\begin{aligned}\left[\eta\frac{d\Phi_m}{dL}\right]_{L_1+} - \left[\eta\frac{d\Phi_m}{dL}\right]_{L_1-} &= -\frac{m\eta_1}{L_1}\Phi_m(L_1) \\ &= \left[\frac{\Phi_0(L_{1-})}{\gamma\tau_a(L_{1-})}e^{-im\varphi-\gamma t} - \frac{im}{L_1^2}\Phi_m(L_1)\right]m\eta_1\delta L_0 e^{im\varphi+\gamma t} \\ &= \frac{m\eta_1}{L_1}\left[\frac{\Omega^2 R_J^2 B_J \delta L_0}{\gamma L_1^2} + \frac{m}{L_1}(L_1 - L_0)\Phi_m(L_1)\right]\end{aligned} \quad (37)$$

Solving (37) for $\Phi_m(L_1)$ gives

$$\Phi_m(L_1) = -\frac{\Omega^2 R_J^2 B_J \delta L_0}{\gamma L_1^2\left[1 + \frac{m}{L_1}(L_1 - L_0)\right]} \quad (38)$$

Note that if I had neglected the $\Phi_0/\gamma\tau_a$ term in (31), that would have resulted in setting the numerator on the right-hand side of (38) equal to zero, which would have implied either $\Phi_m(L_1) = 0$ or $L_1 - L_0 = -L_1/m < 0$, both of which are absurd.

[19] Using (38) in place of (27), we can use the same argument as above ((28) and (29)) to obtain the growth rate for this case

$$\gamma^2 = \frac{m\Omega^2}{1 + \frac{m}{L_1}(L_1 - L_0)} \quad (39)$$

It is interesting to compare this growth rate (39) with that obtained earlier (30) when the acceleration current was ignored. Initially, when $L_1 - L_0 \ll L_1/\text{m}$, (39) gives

$$\gamma \approx \pm\sqrt{m}\,\Omega \quad (40)$$

where, of course, the positive root is the one of interest. This has a weaker dependence on m than the earlier result (30). It may be smaller or larger depending on the ratio

$$\frac{\gamma(\gamma\tau_a \gg 1)}{\gamma(\gamma\tau_a \ll 1)} = \frac{1}{\sqrt{m}}\frac{4\Sigma B_J}{\eta_1\Omega L_1^3} \approx \sqrt{\frac{217}{m}} \quad (41)$$

using the same parameters as before. Thus very small-scale perturbations ($m > \sim 200$) initially grow more slowly as the result of the acceleration current, while larger-scale perturbations ($m < \sim 200$) initially grow faster. The more interesting comparison is in the asymptotic limit as $t \rightarrow \infty$, for which we can assume $L_1 - L_0 \sim L_1$ and (39) gives

$$\gamma \sim \sqrt{\frac{m}{m+1}}\,\Omega \approx \Omega(for \;\; m \gg 1) \quad (42)$$

so the growth rate is limited to the rotation rate, independent of scale size. This is very different from the earlier case (30), where the growth rate itself grows exponentially with time (because L_1 does). This new result (42) is, in fact, the expected behavior if the magnetospheric plasma is constrained to corotate but is otherwise unconstrained by the ionospheric conductivity [*Hill*, 1986; *Vasyliunas*, 1994; *Pontius*, 1997].

3. Conclusion

[20] The approximation $\gamma\tau_a \ll 1$ of subsection 2.1 corresponds to the conventional assumption that the centrifugal drift current is closed entirely by the ionospheric Pedersen current (the acceleration current was assumed to be zero). The opposite approximation $\gamma\tau_a \gg 1$ of subsection 2.2 corresponds to the assumption that the centrifugal drift current is canceled entirely by the acceleration current, so the Pedersen current becomes extraneous (hence the lack of a Σ dependence in (39)). To understand this qualitatively, consider that, if $\gamma \approx \Omega$ (as predicted by (42)), then r, v_r, and dv_r/dt all grow exponentially like $\exp(\Omega t)$, and from this it follows that the acceleration current (third term on the right-hand side of (4)) is equal in magnitude and opposite in direction to the centrifugal drift current (first term on the right-hand side of (4)).

[21] Independent of everything else (e.g., Σ, m, η_1, etc.), the idealized "torus" treated here (a single discontinuous outer edge perturbed by a purely sinusoidal longitudinal perturbation) should eventually have its outer-edge perturbation grow exponentially in time with a growth rate equal to the rotation frequency Ω. In particular, the growth rate should never exceed Ω. The same conclusion was reached by *Vasyliunas* [1994] and *Pontius* [1997] using a simplified form of the ionosphere-magnetosphere coupling equation. I have used the more general form of the ionosphere-magnetosphere coupling equation, but a simplified form of the perturbation. I find, in agreement with Vasyliunas and Pontius, that Ω is an upper limit for γ, and, moreover, that this conclusion applies to the torus itself (where $\gamma\tau_a \sim 1$ for sufficiently small-scale perturbations), as well as to the magnetosphere beyond the torus (where the weaker condition $\Omega\tau_a \sim 1$ prevails).

[22] I have neglected the Coriolis term in the equation of motion. The analysis of *Pontius* [1997] suggests that its inclusion would reinforce our conclusion that $\gamma \leq \Omega$ even in the torus. Both the Coriolis and acceleration terms need to be included in future numerical simulations of the centrifugal interchange instability.

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