Numerical simulation of fine structure in the Io plasma torus 
produced by the centrifugal interchange instability

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The Io plasma torus as a whole has a radial width scale \( \sim 1 R_J \), much larger than the width of the localized Io plasma source (\( \sim 1 R_{Io} \approx R_J/39 \)). One of the most prominent features of the Io torus observed by the Voyager spacecraft and Earth-based instruments is the “ribbon” structure near Io’s orbit. The stability properties of this narrower ribbon structure embedded within the larger torus have been investigated by the Rice Convection Model for Jupiter. Four initial plasma distributions having different radial widths are each represented by 82 longitudinally symmetric edges establishing 41 levels of the flux tube mass content \( \eta \) with the peak \( \eta \) value at Io’s orbit. The same initial perturbation is put on each of these edges and is subjected to centrifugal interchange. Our simulations produce regularly spaced long, thin fingers moving outward from the outer edges. It is shown that the azimuthal width of the interchange convection cells (the distance between outflowing fingers in the nonlinear stage of development) is proportional to the radial width scale of the initial distribution that produced them. The constant of proportionality is \( \approx 0.5 \). Since the exponential growth rate is essentially proportional to the azimuthal wave number of the disturbance and hence is inversely proportional to its azimuthal width, the ribbon-scale interchange structures grow faster than torus-scale interchange structures.


1. Introduction

[2] The Io torus has been probed by both ground-based observations [Trauger, 1984; Schneider and Trauger, 1995] and spacecraft in situ measurements [Thomas et al., 2004, and references therein]. The radial structure of the Io torus can be represented by three concentric components: a hot outer torus (5.9–7 \( R_J \)), a cold inner torus (5.3–5.6 \( R_J \)), and a narrow ribbon-like feature between them. This ribbon-like region is composed of \( \sim 50 \) eV \( S^+ \) and \( S^{2+} \) ions and a cooler component with \( \sim 7–35 \) eV \( S^+ \). A more recent observation during the Cassini encounter [Schneider et al., 2001] also suggested this structure despite substantial torus variability between 1998 and 2000.

[3] The origin of the narrow ribbon structure remains enigmatic. Trauger [1984] argued that the neutral material ejected from Io is ionized in the inner torus to form the ribbon, whereas Hill and Pontius [1998] proposed that the ribbon represents a downstream wake of Io’s localized mass-loading region. In either case, radial transport must be suppressed in the ribbon region in order to explain the persistent presence of the ribbon. Herbert [1996] modeled a ribbon-like feature by using a diffusion formalism to describe the radial transport, and invoking the ring current impoundment mechanism proposed by Siscoe et al. [1981] to drastically reduce the diffusion coefficient in the ribbon region.

[4] The observed radial position of the ribbon exhibits two separate, small but systematic, oscillations: a local time variation that is fixed in the inertial Jovicentric frame and a System III longitude variation that is fixed in the corotating frame [Dessler and Sandel, 1992; Schneider and Trauger, 1995]. A hypothesized large-scale dawn-to-dusk electric field across the magnetosphere can explain the local time variation of ribbon position [Barbosa and Kivelson, 1983; Ip and Goertz, 1983] but not the correlated local time variation of ribbon brightness [Dessler and Sandel, 1992]. Both the local time and longitude variations of ribbon position can be explained if a dawn-dusk electric field is combined with a plasma source that is strongly localized near Io’s orbital position [Smyth and Marconi, 1998]. Like Herbert [1996], Smyth and Marconi [1998] used a diffusion equation to describe radial transport, with a diffusion coefficient that is sharply reduced inside the ribbon. In this paper we neglect the local time and longitude oscillations and focus on the basic problem of radial transport.

[5] Plasma motion in Jupiter’s magnetosphere is driven by the centrifugal force of corotation, not by the solar wind interaction [Hill and Dessler, 1991]. The outflow from the Io torus drives a magnetospheric convection system, which has been described theoretically by three different models: an eddy diffusion model [Siscoe and Summers, 1981; Summers and Siscoe, 1985], a large-scale corotating convection model [Hill et al., 1981], and a transient convection model involving small-scale isolated flux tubes.
The Rice Convection Model has been modified (RCM-J) to numerically simulate plasma transport in the Jovian magnetosphere [Yang et al., 1994]. The magnetospheric plasma is assumed to be confined near the equatorial plane [Hill and Michel, 1976; Siscoe, 1977; Vasyliunas, 1983]. Given an equatorial Io torus plasma distribution $\eta$ (plasma mass per unit magnetic flux), the divergence of the centrifugal drift current is calculated and mapped to Jupiter along magnetic field lines where it is closed through Pedersen currents, thus determining the electric potential $\Phi$. The resulting $E \times B$ drift is then used to advance the equatorial $\eta$ distribution for the next time step. This simulation produced long fingers of outflow from the outer edge of the Io torus interspersed with fingers of inflow from the surrounding magnetosphere.

In this paper, we initialize the system with a toroidal $\eta$ distribution confined to a radial strip of width $\delta$ (full width at half maximum) which has values of 5, 10, 15, and 20 Io radii for the four simulation runs. In order to determine what decides the scale size and shape of the convection cells (fingers), we turn off the velocity shear stabilizing effect [Pontius et al., 1998] and also neglect the ring current impoundment effect [Siscoe et al., 1981]. Thus our purpose is not to produce a realistic model of the ribbon structure, but rather to investigate its intrinsic stability properties in the absence of these known external stabilizing effects. The grid resolution is increased compared to previous RCM-J simulations, and only a limited longitude sector with periodic boundary conditions is simulated in order to resolve the small convection cells physically. By varying the initial radial width $\delta$, we find that the dominant azimuthal-scale size of the fingers is proportional to $\delta$ ($\sim \delta/2$). Our simulation results provide insight into the stability properties of the ribbon structure within the larger torus structure. The large growth rate for this small structure calls for the inclusion of the Coriolis and acceleration currents in future simulations as well as the two known stabilizing mechanisms mentioned above.

2. Rice Convection Model

The Rice Convection Model has been developed and utilized to study solar wind driven convection in Earth’s magnetosphere and its electrodynamic coupling to the ionosphere [Toffoletto et al., 2003, and references therein]. It is based on the logic scheme proposed by Vasyliunas [1970]. The RCM deals with the Earth’s inner and middle magnetosphere where the plasma flow speed is much less than the fast mode speed [Wolf, 1983]. This model has been adapted (the RCM-J) to treat rotationally driven convection in Jupiter’s magnetosphere [Yang et al., 1994; Pontius et al., 1998], where curvature gradient drift currents are replaced by centrifugal drift current [Hill, 1983]. The plasma sheet integrated magnetospheric current density is given by equation (4) of [Hill, 1983], namely

$$J_\perp = \frac{\eta \left[ \frac{1}{2} r^2 \dot{\phi} + 2 \Omega v - \hat{\mathbf{z}} \times (\text{d}v/\text{d}t) \right]}{B}$$

where

$$\eta = \int \rho \text{d}z/B$$

is the flux tube mass content. The three terms on the right hand side of equation (1) are the centrifugal, Coriolis, and acceleration currents respectively, which are of zero, first, and second order in the ratio $v/\Omega r$. The divergence of this equatorial current is balanced by the field-aligned Birkland current density into the equatorial plane from both northern and southern ionospheres,

$$f_{ij} = -\nabla_e \cdot \mathbf{J}_\perp$$

where $\nabla_e$ is the two-dimensional gradient operator in the equatorial plane. The equation of conservation of current in the ionosphere is

$$\nabla_h \cdot \left( \Sigma \cdot (-\nabla_h \Phi) \right) = f_{ij} \sin(I)$$

where $\nabla_h$ is the horizontal gradient operator in the ionosphere, $\Sigma$ is the ionospheric conductance tensor for both hemispheres, and $\Phi$ is the electrostatic potential. $I$ is the magnetic dip angle. $f_{ij}$ is the current density along the field line from the magnetosphere which is related to $f_{ij}$ by

$$\frac{f_{ij}}{B_i} = \frac{f_{ij}}{B_j}$$

Under the assumption that the magnetic field doesn’t depend on time, $E$ is derivable from $\Phi$. With the ideal MHD approximation

$$E + v \times B = 0$$

a closed set of equations (1)–(6) can be solved simultaneously and self consistently. The resulting $E \times B$ drift is then used to advance the equatorial $\eta$ distribution for the next time step.

We use the same magnetic field model (a spin-aligned dipole) and the same (uniform) ionospheric conductance model as described by Yang et al. [1994]. The essential differences between our new simulations and previous ones are that we decrease the size of the simulation region and increase numerical grid resolution, and we limit the initial plasma distribution to a thin radial region ($5-20 R_J$) localized near Io’s orbit, not the whole torus region ($\sim 1 R_J$). We turn off the velocity shear stabilizing effect as described by Pontius et al. [1998], and we neglect the gradient curvature drift current. By limiting our simulation to the early stage of finger development ($v < \Omega r$), we consider only the first term in equation (1) and neglect the second and third terms that are of first and second order in $v/\Omega r$, respectively.

3. Simulation Setup

We set up the simulation in a region of Jupiter’s equatorial plane with $0 \leq \phi \leq 30^\circ$ and $5.51 \leq L \leq 6.51$, where $\phi$ is the longitude angle and $L$ is the Jovicentric distance of the equatorial crossing point of a dipole field line in units of Jupiter’s radius $R_J$. This equatorial region is mapped along dipole magnetic field lines into the ionosphere in a strip of colatitudes $23.1^\circ \leq \theta \leq 25.2^\circ$. This simulation region is shown in Figure 1. The boundary
conditions are $\Phi = \text{const}$ (Dirichlet) at $L = 5.51$, $\frac{\partial \Phi}{\partial n} = 0$ (Neumann) at $L = 6.51$, and $\Phi(\phi = 0) = \Phi(\phi = 30^\circ)$. It will become apparent in the figures below that the boundary conditions have negligible effect on the form of the solution.

[10] We initialized the system with a toroidal cold $\eta$ distribution confined to a radial strip of width $\delta$. We neglect the $\lambda$III asymmetry of the ribbon structure [Dessler and Sandel, 1992] and put the center of this distribution at Io’s orbit with maximum $\eta$ there. We use the following simple distribution function:

$$
\eta(r) = \begin{cases} 
\eta_{\text{max}} \cos^2 \frac{\pi (r - r_{\text{Io}})}{2 \delta} & : |r - r_{\text{Io}}| \leq \delta \\
0 & : \text{elsewhere}
\end{cases}
$$

(7)

where $\eta_{\text{max}} = 2.6 \times 10^{-3}$ kg/Wb is the peak $\eta$ value at Io’s orbit, and $r_{\text{Io}} = 5.91 \, R_J$ is the radius of Io’s orbit. This function restricts the plasma distribution to a region symmetric about Io’s orbit with radial width $2\delta$.

[11] We used the traditional edge-based RCM-J algorithm [Yang et al., 1994]. The spatial variations of $\eta$ are represented by 82 edges (41 outer edges and 41 initially symmetric inner edges). Each pair of outer and inner edges have the same $\eta$ level, resulting in 41 $\eta$ levels. The distribution function (7) and the radial arrangement of these 41 edges are shown in Figure 2.

[12] We apply an initial perturbation to each of the outer edges, which are unstable under centrifugal interchange because the plasma content $\eta$ decreases with distance there [Huang and Hill, 1991]. The initial perturbation is specified as

$$
\Delta I = 0.01 \times \sum_{m=-1}^{M} \frac{1}{m} \sin(m\phi - \varphi(m))
$$

(8)

where $I$ is the latitudinal grid index, $M = 50$, and the phase $\varphi(m)$ comes from a random number generator.

4. Simulation Results

4.1. Finger Evolution

[13] Figure 3 shows the development of the interchange fingers during one of our simulations. For clarity, we plot only 12 edges (constant $\eta$ levels), including the outermost and innermost edges. The initial perturbation is too small to be visible in Figure 3a, but by $t = 79$ min (Figure 3b) some small ripples are clearly evident on the outer edges. These ripples grow into fingers and approach the outer boundary by $t = 110$ min (Figure 3d), when the simulation was stopped. The distorted shape results from the fringing electric field near the tips of the fingers [Thomas et al., 2004].
4.2. Grid Convergence Study

[14] Our first series of runs is a grid convergence study to determine how many grid points in our simulation region are needed to resolve the convection cells correctly. In these runs, we keep the initial plasma distribution width (δ = 10 \( R_{Io} \)) and the simulation region (30° in longitude) fixed, but change the number of grid points in the simulation region from 100 to 600. The resulting finger configurations are shown in Figure 4. We choose the times when the finger developments are roughly comparable for different grid spacings for display. The initial perturbation is the same for all six runs. Thus the obvious difference between results a, b, and c is a result of the different grid resolutions, while the virtually identical appearance of results e and f indicates that adequate grid resolution has been achieved.

[15] Our ability to resolve the physical convection cells (fingers) depends on the dimensionless ratio (grid cell size)/δ. This dependence is illustrated in the summary plot shown in Figure 5, on the basis of the six runs shown in Figure 4. When the grid cell size is large (right side of the plot), the finger spacing λ varies more or less linearly with grid size. As the grid size decreases, the finger spacing levels off at a value 0.44δ, independent of grid spacing, which we take to be the physical-scale size. To reach this limit, however, the grid spacing itself has to be smaller by an additional factor \( \sim 10 \), that is 0.04δ, which is achieved by using more than 300 grid points in the longitudinal section. In our following runs, we use 400 \( \times \) 400 grid points in the simulation region, which means that for each finger, we have almost 15 grid points to resolve it. These fingers are physical results, not depending on grid cell size.

4.3. Effect of the Initial Radial Width on Finger Spacing

[16] Fixing the simulation region with 400 \( \times \) 400 grid points, we carried out four additional runs, using δ values 5 \( R_{Io} \), 10 \( R_{Io} \), 15 \( R_{Io} \), and 20 \( R_{Io} \) for the initial radial distribution. The same set of random numbers is used for each case. The four resulting finger configurations are shown in Figure 6. Comparing the four plots of Figure 6 indicates that when the initial radial width is smaller, the resultant fingers are more dense in their longitudinal spacing; that is, the average distance between the fingers (and their average azimuthal width) is smaller. Because the fingers have a spectrum of heights, a manual count of the fingers would be ambiguous. Thus we performed a Fast Fourier transform (FFT) to determine the dominant frequency in the finger waveforms, then determined the number of fingers and the spacing between fingers from the FFT results. The results are shown in Table 1.

[17] The results in Table 1 are summarized in Figure 7. The four data points are reasonably fit by the straight line with the equation

\[
\lambda = 0.523 \times \delta
\]
with an uncertainty of ±0.026 in the slope. That is, the dominant azimuthal width scale of the interchange convection cells (fingers) in their nonlinear stage of development is proportional to the radial width scale of the initial distribution that produced them. The constant of proportionality is \( rC^{0.5} \).

5. Discussion

[18] Earlier RCM-J results [Yang et al., 1994; Pontius et al., 1998] were significantly affected by the grid spacing, which was not always adequate to resolve the physical-scale size implied by the initial conditions. Our simulations of fine structures show that the final finger configuration is independent of grid spacing only when we use at least 300 grid points in a 30° longitudinal section. This corresponds to about 15 grid points per finger width.

[19] The radial width of the initial distribution is the only factor we changed during our second series of runs. These results show a linear dependence of finger spacing on the radial width of the initial distribution. The earlier simulations of the large-scale torus also showed that the finger-scale size depends on the radial density gradient of the initial torus [Yang et al., 1994], but these results were also affected by the grid resolution. Our simulations of fine structures converge to the physical results when we use 400 grid points in a 30° longitudinal section. The initial plasma distribution is then the only factor determining the finger-scale size.

[20] Yang et al. [1994] suggested that the outward flow of plasma in the fingers is the primary mechanism of plasma transport from Io’s orbit to the outer magnetosphere by the centrifugal interchange instability. An analytical linear analysis Huang and Hill [1991] shows that the exponential growth rate, for a given value of \( \eta_{max} \) and \( \Sigma \), is proportional to the azimuthal wave number of the disturbance, and hence inversely proportional to its azimuthal-scale size. (We find, in agreement with previous RCM-J results, that the growth rate for a given wave number scales with ratio \( \eta_{max}/\Sigma \), as predicted by the linear theory.) Since the azimuthal width of the convection cells is proportional to the radial width of the initial distribution, the small (ribbon-scale) structures grow faster than the larger (torus-scale) structures. For small-scale structures, the restriction \( v < \Omega r \) is violated quickly. Using \( \delta = 10 R_{Io} \), the convection speed of the longest finger’s tip at \( t = 87 \) min is \( 10^{-3} R/s \), which is comparable to the corotation speed of this tip (\( \Omega r = 1.1 \times 10^{-3} R/s \)). When the radial transport speed becomes comparable to the rotation speed, the acceleration current becomes the primary mechanism for closure of the centrifugal drift current [Hill, 2006]. We would then need to take the second and third terms in equation (1) into account. These two terms, the Coriolis and acceleration currents, will be included in our future simulations of the Io torus.

Figure 3. Finger evolution of the initial perturbations for an initial distribution width \( \delta = 10 R_{Io} \) at four time steps in the equatorial plane. (a) Initial configuration (all 82 edges) is shown. (b) Small ripples become clearly visible by \( t = 79 \) min and (c and d) continue to grow into fingers, approaching the outer boundary at \( t = 110 \) min.
Figure 4. Finger configurations when using different numbers of grid points in the 30° longitudinal section: (a) 100 grid points at $t = 142$ min, (b) 200 grid points at $t = 123$ min, (c) 300 grid points at $t = 113$ min, (d) 400 grid points at $t = 110$ min, (e) 500 grid points at $t = 110$ min, and (f) 600 grid points at $t = 110$ min.
Figure 5. Results of grid convergence study.

Figure 6. Final finger configuration for four different $\delta$ values: (a) $\delta = 5 \, R_{Io}$, (b) $\delta = 10 \, R_{Io}$, (c) $\delta = 15 \, R_{Io}$, and (d) $\delta = 20 \, R_{Io}$.
We emphasize that we have not attempted to produce a realistic model of the ribbon structure. In addition to neglecting the local time and longitude variations, we have deliberately turned off the two known stabilizing effects (ring current impoundment and velocity shear) that are, at least in part, responsible for maintaining the ribbon as a coherent structure. By neglecting these complications, we have been able to focus on the factor that determines the size scale of the dominant convection cells, namely, the width of the initial radial distribution. We have shown that the dominant azimuthal-size scale of the convection cells is about 1/2 of the radial width of the initial distribution. Thus ribbon-scale perturbations \((\sim R_{\text{Io}})\) grow much faster than torus-scale perturbations \((\sim R_J)\) for given values of the flux tube mass content and ionosphere conductance. This result emphasizes the importance of stabilizing mechanisms for maintaining the torus structure in general, and the ribbon structure in particular. The two known stabilizing mechanisms (ring current impoundment and velocity shear) can and will be included in future RCM-J simulations.

In agreement with previous RCM-J simulations, we have found that the radial transport process driven by a given radially confined plasma source cannot be described by a radial diffusion equation (i.e., a random walk of flux tubes in their radial “L” coordinate). Our results do, however, support the conventional assumption of diffusion-based models [e.g., Richardson and Siscoe, 1981; Herbert, 1996; Smyth and Marconi, 1998] that the azimuthally averaged rate of outward mass transport outside the torus greatly exceeds that of inward mass transport inside the torus.

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Figure 7. Linear relationship between finger spacing and initial radial width. The error bars of the finger spacing come from the uncertainty of the peak location in the FFT spectrums.

Table 1. Relation Between Finger Spacing and Initial Distribution Width at Io’s Orbit

<table>
<thead>
<tr>
<th>( \delta, R_{\text{Io}} )</th>
<th>Simulation Region</th>
<th>Grid Points</th>
<th>Number of Fingers</th>
<th>Finger Spacing ( \lambda, R_J )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>( R_J, 30^\circ )</td>
<td>400 \times 400</td>
<td>42</td>
<td>0.0737</td>
</tr>
<tr>
<td>10</td>
<td>( R_J, 30^\circ )</td>
<td>400 \times 400</td>
<td>27</td>
<td>0.1146</td>
</tr>
<tr>
<td>15</td>
<td>( R_J, 30^\circ )</td>
<td>400 \times 400</td>
<td>16</td>
<td>0.1934</td>
</tr>
<tr>
<td>20</td>
<td>( R_J, 30^\circ )</td>
<td>400 \times 400</td>
<td>12</td>
<td>0.2579</td>
</tr>
</tbody>
</table>

*The number of fingers has an uncertainty of ±1 resulting from the FFT spectrum.


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